Predicting the constitutive behaviour of microsphere-filled elastomers: the influence of shell buckling and interaction

William J. Parnell
Professor of Applied Mathematics and EPSRC Research Fellow
School of Mathematics, University of Manchester, UK

Innovations in Rubber Design
7th December 2016
work with R. De Pascalis, M. Thorpe, G. Wyn Jones and I.D. Abrahams
Overview

- Background
  - WICC - my research group
  - Problem of interest - elastomeric composites
Overview

- Background
  - WICC - my research group
  - Problem of interest - elastomeric composites

- Constitutive modelling of microsphere filled elastomers (syntactic foams)
  - nonlinear load curve
Overview

- Background
  - WICC - my research group
  - Problem of interest - elastomeric composites

- Constitutive modelling of microsphere filled elastomers (syntactic foams)
  - nonlinear load curve
  - microsphere buckling models
Overview

- **Background**
  - WICC - my research group
  - Problem of interest - elastomeric composites

- Constitutive modelling of microsphere filled elastomers (syntactic foams)
  - Nonlinear load curve
  - Microsphere buckling models

- Wave propagation in pre-stressed microsphere composites
Overview

- Background
  - WICC - my research group
  - Problem of interest - elastomeric composites

- Constitutive modelling of microsphere filled elastomers (syntactic foams)
  - nonlinear load curve
  - microsphere buckling models

- Wave propagation in pre-stressed microsphere composites

- Perspectives
Overview

- **Background**
  - WICC - my research group
  - Problem of interest - elastomeric composites

- **Constitutive modelling of microsphere filled elastomers (syntactic foams)**
  - nonlinear load curve
  - microsphere buckling models

- **Wave propagation in pre-stressed microsphere composites**

- **Perspectives**
  - More general constitutive modelling
Overview

- Background
  - WICC - my research group
  - Problem of interest - elastomeric composites

- Constitutive modelling of microsphere filled elastomers (syntactic foams)
  - nonlinear load curve
  - microsphere buckling models

- Wave propagation in pre-stressed microsphere composites

- Perspectives
  - More general constitutive modelling
  - Metamaterials and tunable materials
Overview

- **Background**
  - WICC - my research group
  - Problem of interest - elastomeric composites

- **Constitutive modelling of microsphere filled elastomers**
  (syntactic foams)
  - nonlinear load curve
  - microsphere buckling models

- **Wave propagation in pre-stressed microsphere composites**

- **Perspectives**
  - More general constitutive modelling
  - Metamaterials and tunable materials
  - Microsphere composites project (opportunities)
- Background
- A traditional modelling problem (with Thales)
- Some wave concepts
- Waves in pre-stressed media
- Tunable band-gaps
WICC - Waves in Complex Continua

- Group founded in Sept 2010, led by Parnell and Abrahams (now Director of INI, Cambridge)
WICC - Waves in Complex Continua

- Group founded in Sept 2010, led by Parnell and Abrahams (now Director of INI, Cambridge)
- General objective is to model, mathematically, complex materials
WICC - Waves in Complex Continua

- Group founded in Sept 2010, led by Parnell and Abrahams (now Director of INI, Cambridge)
- General objective is to model, mathematically, complex materials
- Includes general constitutive modelling as well as wave propagation phenomena
WICC - Waves in Complex Continua

- Group founded in Sept 2010, led by Parnell and Abrahams (now Director of INI, Cambridge)
- General objective is to model, mathematically, complex materials
- Includes general constitutive modelling as well as wave propagation phenomena
- Large group, around 25 members
WICC - Waves in Complex Continua

- Group founded in Sept 2010, led by Parnell and Abrahams (now Director of INI, Cambridge)
- General objective is to model, mathematically, complex materials
- Includes general constitutive modelling as well as wave propagation phenomena
- Large group, around 25 members
- Breadth of funding from EPSRC, Royal Society, Leverhulme Trust and Industry (Dyson, Thales, NNS)
Research interests, www.wiccwavesgroup.weebly.com

- Inhomogeneous media models
- Industrial modelling
- Phononics, metamaterials
- Biological tissues

- www.pump-zone.com
- www.femto-st.fr
- Coutinho et al (2007)
Background to Manchester and my research group

Constitutive modelling of microsphere filled elastomers

Wave propagation in pre-stressed microsphere composites

Perspectives
Microsphere composites (syntactic foams)

SEM image (a) Hollow Glass Microsphere, (b) surface/shell structure. Li et al. [2011].
Microsphere composites (syntactic foams)

SEM image (a) Hollow Glass Microsphere, (b) surface/shell structure. Li et al. [2011].

Reasons to use microspheres in matrix composites:

- Add volume whilst reducing weight
- Improve thermal and sound insulation
- Increase compressibility and reduce cost

Isotropic, homogeneous rubber matrix filled with expancel (glassy) microspheres - distribution of shell thicknesses.
Industrial interest and relation to waves

Need to understand how the material responds \textit{acoustically} when it is under significant pre-stress.

Shorter et al. (2008)
Industrial interest and relation to waves

Need to understand how the material responds \textit{acoustically} when it is under significant pre-stress.

Shorter et al. (2008)

To understand the acoustic response we need a model for the constitutive behaviour of the medium (strongly related).
Motivation

Need to be able to predict the so-called pressure-volume curve associated with this material
Motivation

Need to be able to predict the so-called pressure-volume curve associated with this material

Experiments very difficult and costly to perform
Motivation

Need to be able to predict the so-called pressure-volume curve associated with this material

Experiments very difficult and costly to perform

Repeatability is difficult
Motivation

Need to be able to predict the so-called pressure-volume curve associated with this material.

Experiments very difficult and costly to perform.

Repeatability is difficult.

Dependence on a number of parameters can be tested with the model.
Motivation

Need to be able to predict the so-called pressure-volume curve associated with this material

Experiments very difficult and costly to perform

Repeatability is difficult

Dependence on a number of parameters can be tested with the model

Require careful models incorporating all of the major effects
Geometry and assumptions

In practice the material response is strongly nonlinear - why?
Geometry and assumptions

In practice the material response is strongly **nonlinear** - why?

We need a model for the **nonlinear constitutive behaviour** of the medium.
Geometry and assumptions

In practice the material response is strongly nonlinear - why?
We need a model for the nonlinear constitutive behaviour of the medium.
Single shell - model - dilute distribution
We need a relationship

\[ \delta V = \delta V(p) \]

\[ \delta V = \text{relative volume change.} \]
We need a relationship

\[ \delta V = \delta V(p) \]

\[ \delta V = \text{relative volume change.} \]

\( p < p_c \)
- **Pre-buckling**
- Linear elasticity

\( p = p_c \)
- **Buckling**
- Fok-Allwright model

\( p > p_c \)
- **Post-buckling**
- Nonlinear elasticity

\( \delta V(p) \) is then integrated response over all shell thicknesses.
Parameter studies and some plots

Rubbery matrix. Glassy expancel microspheres. Initial volume fraction of microspheres = 5%, Gamma distribution of shell thicknesses.

Neo-Hookean (dotted), Mooney-Rivlin (solid), nearly-compressible (dashed) and linear elasticity (dot-dash).

De Pascalis, R., WJP and Abrahams, I.D., 2013, JMPS
Investigated shell buckling inside elastomers. General method derived - energy minimization. **Stiff, thin-shell limit** derived:

Hydrostatic buckling pressure of a thin, glassy shell \( p \) relative to the Classical buckling pressure for an unembedded shell, 

\[
p_0 = \frac{4\mu_s(1+\nu_s)}{\sqrt{3(1-\nu_s)}} \]  

(Zoelly-Van der Neut), for each buckling mode \( n \).
Background to Manchester and my research group
Constitutive modelling of microsphere filled elastomers
Wave propagation in pre-stressed microsphere composites
Perspectives
Other inspiration

Inhomogeneous soft materials occur frequently in natural and synthetic forms:

- Polymer based composites
- Self-assembly based hydrogels
- Soft tissue
Other inspiration

**Inhomogeneous** soft materials occur frequently in natural and synthetic forms:

- Polymer based composites
- Self-assembly based hydrogels
- Soft tissue

Their dynamic response in pre-stressed states is fundamental.
Other inspiration

Inhomogeneous soft materials occur frequently in natural and synthetic forms:

- Polymer based composites
- Self-assembly based hydrogels
- Soft tissue

Their dynamic response in pre-stressed states is fundamental.

Difficulties:

- Homogeneous deformations are not possible
Other inspiration

Inhomogeneous soft materials occur frequently in natural and synthetic forms:

- Polymer based composites
- Self-assembly based hydrogels
- Soft tissue

Their dynamic response in pre-stressed states is fundamental.

Difficulties:

- Homogeneous deformations are not possible
- Microstructure evolution
Other inspiration

Inhomogeneous soft materials occur frequently in natural and synthetic forms:

- Polymer based composites
- Self-assembly based hydrogels
- Soft tissue

Their dynamic response in pre-stressed states is fundamental.

Difficulties:

- Homogeneous deformations are not possible
- Microstructure evolution
- Changes in material properties
Other inspiration

**Inhomogeneous** soft materials occur frequently in natural and synthetic forms:

- Polymer based composites
- Self-assembly based hydrogels
- Soft tissue

Their dynamic response in pre-stressed states is fundamental.

**Difficulties:**

- Homogeneous deformations are not possible
- Microstructure evolution
- Changes in material properties
- Response is often strongly frequency dependent
Wave scattering in pre-stressed microsphere composites

Modify the classical Multiple Scattering Theory of Mal and Bose (1974) as mechanism for evaluating effective wavenumber.
Wave scattering in pre-stressed microsphere composites

Modify the classical Multiple Scattering Theory of Mal and Bose (1974) as mechanism for evaluating effective wavenumber. E.g.

\[ k_\star^s = F(\phi, A_2, A_1) \rightarrow k_\star^s(p_{\infty}) = f(\phi(p_{\infty}), a_2(p_{\infty}), a_1(p_{\infty})) \]
Wave scattering in pre-stressed microsphere composites

Modify the classical Multiple Scattering Theory of Mal and Bose (1974) as mechanism for evaluating effective wavenumber. E.g.

\[ k_s^* = F(\phi, A_2, A_1) \rightarrow k_s^*(p_\infty) = f(\phi(p_\infty), a_2(p_\infty), a_1(p_\infty)) \]

which gives the effective (incremental) shear modulus at fixed pre-stress

\[ \mu_* = g(\mu_1/\mu_0, \phi, p_\infty) \]
Background to Manchester and my research group
Constitutive modelling of microsphere filled elastomers
Wave propagation in pre-stressed microsphere composites
Perspectives
General constitutive modelling

Significant interest in nonlinear viscoelasticity,
General constitutive modelling

Significant interest in nonlinear viscoelasticity, e.g. QLV

\[ \Pi(t) = \int_{-\infty}^{t} G(t - s) \frac{d\Pi^e}{ds}(C(s)) \, ds \]
General constitutive modelling

Significant interest in nonlinear viscoelasticity, e.g. QLV

$$\Pi(t) = \int_{-\infty}^{t} G(t - s) \frac{d\Pi^e}{ds} (C(s)) \, ds$$

and extensions to accommodate strain dependent relaxation

$$\Pi(t) = \int_{-\infty}^{t} G(t - s, C) \frac{d\Pi^e}{ds} (C(s)) \, ds$$
General constitutive modelling

Significant interest in nonlinear viscoelasticity, e.g. QLV

\[ \Pi(t) = \int_{-\infty}^{t} G(t - s) \frac{d\Pi^e}{ds}(C(s)) \ ds \]

and extensions to accommodate strain dependent relaxation

\[ \Pi(t) = \int_{-\infty}^{t} G(t - s, C) \frac{d\Pi^e}{ds}(C(s)) \ ds \]

and specifically motivated by microstructural models, for e.g. tendon, filled polymers, etc. again informed by imaging.

Metamaterials & tuneable structures


Also interest in tuning band structure of periodic elastomers via pre-load, or perhaps more interestingly retain them under pre-load (invariance) ??

See poster by Zhang and Parnell tomorrow
Microsphere modelling project

Two year project as part of my fellowship

- 2 year PDRA - theoretical modelling (currently advertised)
- 2 year PDRA - XCT imaging (advertised shortly)

Image-based modelling, particularly regarding behaviour under load

[T. Lowe & S. Coban, Henry Moseley Centre, Manchester]
Acknowledgements

- EPSRC (Parnell “NEMESIS” Fellowship EP/L018039/1)
- EPSRC (De Pascalis PDRA EP/H050779/1)
- Thales UK/EPSRC (Thorpe CASE PhD)
- Royal Society (Abrahams Wolfson)