Dynamic Systems with Rubber Contacts in Technical Applications

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- Introduction
- Adhesion experiments
- Modelling of dynamic systems with elastomer contacts
- Comparison with experiments
- Conclusion
Examples of systems with elastomers

- Wiper Window seal
- Tread block
- Brake booster
- Tyre
- Shock absorber
Experimental investigation of adhesion

Adhesion test rig

- Electrodynamic shaker applies a preload $F_{\text{pre}}$ on the contact pairing (rubber/grinded steel ball)
- Contact partners are rapidly separated after a defined preload duration $t_{\text{pre}}$

Steel ball: diameter 10 mm
Rubber sample: thickness 6 mm, diameter 10 mm
Experimental investigation of adhesion

Basic tests

- All experiments are performed with the same rubber sample
- Cleaning with acetone-moistened towel
- Relaxation behaviour is observed
- Adhesion effect vanishes if plastic film is applied

- Repeatability of experiment is very good
Experimental investigation of adhesion

**Variation of test parameters**

- Adhesion peak force strongly increases for $t_{\text{pre}} < 20s$, then approx. constant
- Good approximation with exponential function [Roberts and Thomas 1975]
- Area of real contact augments with time and rubber stiffness decreases

- Adhesion peak force increases with increasing preload
- Approximation by a power law

$$F_{\text{max}} = b_0 F_{\text{pre}}^{b_1} \quad \text{with } b_1=0.57 \quad \text{[Hertz, Schallamach: } b_1=0.66]\)
Modelling of dynamic systems

Steady state simulations
- Highly sophisticated 3D models
- Larger number of DOFs
- Non-linear material description
- Thermo-mechanical coupling
- Numerically very expensive

Dynamics simulations
- Simple models (1 to 3 DOF)
- Neglect of geometry and structural effects
- Fast calculation
- Mostly add-ons for tyre models

Aim of the presented approach: Development of an numerically efficient model for the high-frequency dynamics of single tread blocks
Modelling of dynamic systems

Predefined kinematics

Module 1: Structural dynamics

Local friction forces

Module 2: Local friction characteristic

Local contact deformations

Module 3: Non-linear contact stiffness

Module 4: Local wear process

Nodal displacements

Local normal forces

Module 5: Adhesion
Tyre tread block as example

- Separate modelling of structural dynamics and contact mechanics
- Interaction of modules during simulation

Module 1: Structural dynamics

Module 2: Local friction characteristic

Module 3: Non-linear contact stiffness

Module 4: Local wear process

Module 5: Adhesion

Contact with rough surface

Modelling as contact of smooth surfaces
Modelling of dynamic systems

Efficient description of structural dynamics (Craig/Bampton-reduction)

\[
\begin{bmatrix}
M_{NN} & M_{NH} \\
M_{HN} & M_{HH}
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_N \\
\ddot{u}_H
\end{bmatrix}
+ \begin{bmatrix}
D_{NN} & D_{NH} \\
D_{HN} & D_{HH}
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_N \\
\ddot{u}_H
\end{bmatrix}
+ \begin{bmatrix}
K_{NN} & K_{NH} \\
K_{HN} & K_{HH}
\end{bmatrix}
\begin{bmatrix}
u_N \\
\nu_H
\end{bmatrix} = \begin{bmatrix}
f_N \\
f_H
\end{bmatrix}
\]

FE system equation

DOF reduction by use of reduced coordinates

\[u \approx T_{\text{red}} \bar{u}_{\text{red}}\]

\[
\begin{bmatrix}
T_{\text{red}}^T M_{\text{red}} T_{\text{red}} \\
T_{\text{red}}^T D_{\text{red}} T_{\text{red}} \\
T_{\text{red}}^T K_{\text{red}} T_{\text{red}}
\end{bmatrix} \begin{bmatrix}
\ddot{u}_{\text{red}} \\
\ddot{\nu}_{\text{red}} \\
\dddot{u}_{\text{red}}
\end{bmatrix} + \begin{bmatrix}
T_{\text{red}}^T f_{\text{red}}
\end{bmatrix} = T_{\text{red}}^T f
\]

Static shape functions

Modal shape functions
Determination of local friction characteristic

- Experiments
- Analytical descriptions
- Simulations

Approximation function:

\[ \mu_i(v_{rel,i}, p_{N,i}) = \frac{2}{\pi} \arctan \left( k_s v_{rel,i} \right) \left( \mu_{\infty,v} \mid (\mu_{0,v} \quad \mu_{\infty,v}) \cdot \left( \gamma_v \left| v_{rel,i} \right| \right) \right) \]

- Smoothing term
- Influence of velocity
- Influence of pressure
Modelling of dynamic systems

Determination of local friction characteristic

- Experiments
- Analytical descriptions
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**Approximation function:**

\[
\mu_i(v_{rel,i}, p_{N,i}) = \frac{2}{\pi} \arctan \left( k_S v_{rel,i} \right) \left( \mu_{\infty,v} \mid (\mu_{0,v} \quad \mu_{\infty,v}) \right) \left( \gamma_v \mid v_{rel,i} \right) \cdot \left( \mu_{\infty,p} \mid (\mu_{0,p} \quad \mu_{\infty,p}) \right) \left( \gamma_p \mid p_{N,i} \right)
\]

- Smoothing term
- Influence of velocity
- Influence of pressure

Experimente: [Gäbel 2009]
Modelling of dynamic systems

Investigation of non-linear force-displacement characteristic

\[ F_N = \begin{cases} 30 \text{ N} \\ 60 \text{ N} \\ 90 \text{ N} \\ 120 \text{ N} \end{cases} \]

Few junctions:
Large increase of local pressure → high deformations → small contact stiffness

Many junctions
Relatively smaller increase of local pressure → small deformations → high stiffness

\[ c_N = \frac{\Delta F_N}{\Delta s_N} \]

[\text{Gäbel 2009}]
Modelling of non-linear contact stiffness

- Local non-linear contact stiffness $c_{c,i}$ at each local contact element:

$$c_{c,i}(u_i) = c_\infty \left(1 - e^{-ku_i}\right)$$

- Contact algorithm similar to Penalty approach
- Good agreement between experiment and simulation for corundum and concrete surface

Normal force vs. normal displacement

**Experiments:** [Gäbel 2009]
Modelling of non-linear contact stiffness

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Contact stiffness vs. normal displacement

Experiments: [Gäbel 2009]

Contact stiffness vs. normal displacement

Experiments:
Experimental stick-slip investigation

Tread block on corundum surface grit 400

\[ v = 300 \text{ mm/s} \]
\[ p_N = 0.1 \text{ N/mm}^2 \]

Measured displacement

Measured velocity

Measured limit cycle

\[ f_{ss} = 1540 \text{ Hz} \]
Comparison with simulation

- Friction characteristic and non-linear contact stiffness from experiments on corundum
- Elasticity modulus $E=22 \text{ N/mm}^2$
- Damping coefficient $\beta=2.8 \times 10^{-4} \text{s}^{-1}$
- Stick-slip frequency and displacement amplitude show good agreement
- Slight differences at transition from sliding to sticking phase
Tread block is still modelled with fixed support
Influence of tread block dimensions

- Nominal pressure $p_N = 0.25 \text{ N/mm}^2$
- Sliding velocity $v_0=2000 \text{ mm/s}$
- Reduction of block height leads to reduction of edge effects
- Reduction of pressure peak at run-in increases tread block friction coefficient
- Increase of block length leads to homogenised pressure distribution
- Increase of shear stiffness increases tread block friction coefficient
Influence of local wear

Change of geometry due to wear

- Initial nominal normal pressure $p_N = 0.25 \text{ N/mm}^2$
- Sliding velocity $v_0 = 2000 \text{ mm/s}$
- Pronounced wear at run-in edge
- Pressure distribution homogenises with sliding distance
- Global tread block friction coefficient increases with wear
Conclusion

- Adhesion test rig allows repeatable pull-off separation tests
- Higher preload levels and longer preload durations enlarge the adhesion peak force
- Modular model considers structural dynamics, parameter dependent friction, non-linear contact stiffness and wear
- Craig/Bampton transformation allows efficient calculation of the tread block dynamics
- Rough surface is modelled as smooth with respective non-linear contact stiffness and local friction characteristic
- Experimentally identified non-linear friction characteristic and wear law are implemented
- Good agreement between stick-slip experiments and simulation

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